Distributed piezoelectric sensors for boundary forces measurements

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Abstract

The identification of boundary quantities such as bending moments and/or shear forces are not easy because

- of their accessibility;
- they are proportional to the spatial derivatives of the displacements (sensitive to errors in measurements).

This work introduces a new class of distributed piezoelectric sensors based on the laminate theory and the weak form of equation of motion of beams. By this way, the extraction of bending moments or shear forces are obtained at boundaries thanks to the spatial integrations made by piezoelectric patches (or PVDF films) with particular shapes.

Mathematical principles of the shear force extraction

The studied mechanical system is a Flexural Euler-Bernoulli beam. Considering a forced harmonic vibration, the motion equation can be written as follows :

$$EI\frac{\partial^4 w}{\partial x^4} - \mu\omega^2 w = F \qquad (1)$$

The weak form of the (1) on a region without excitation is:

$$\int_0^a \frac{\partial^2 \Psi}{\partial x^2}\left[EI\frac{\partial^4 w}{\partial x^4} - \mu\omega^2 w\right]dx = 0 \qquad (2)$$

where ψ and is a test function.

By choosing the test function:

$$\Psi(x) = \frac{1}{2}x^2 - 5x^4/a^2 + 10x^5/a^3 - \frac{15}{2}x^6/a^4 + 7x^7/a^5$$

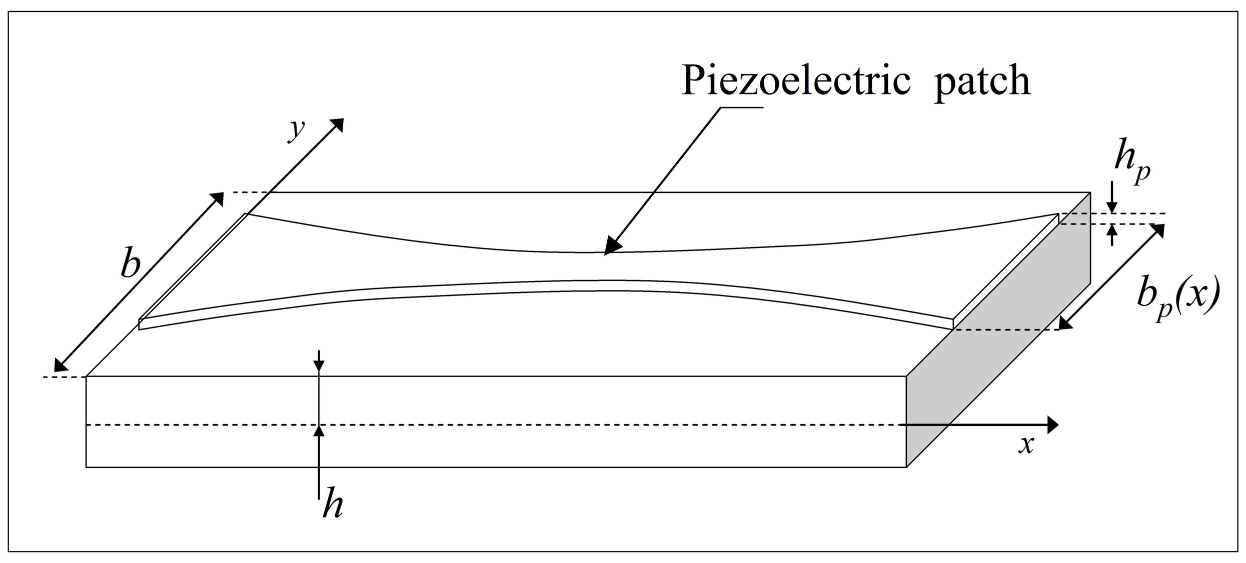
it is possible to extract the exact expression of the shear force at x=0:

$$\Lambda(0) = -\int_0^a \frac{\partial^2 w}{\partial x^2}\left[EI\frac{\partial^4 \Psi}{\partial x^4} - \mu\omega^2\Psi\right]dx \qquad (3)$$

Shape of the piezoelectric sensor

A distributed PVDF sensor of width $b_p(x)$ is considered. A charge amplifier connected to the sensor delivers:

$$Q(t) = -d_{31}c^E h\int_0^a \frac{\partial^2 w}{\partial x^2}(x,t)b_p(x)dx$$



By choosing a sensor shape linked to the test function and its derivatives, the boundary shear force can be directly measured by:

$$\Lambda(0) = -\int_0^a \frac{\partial^2 w}{\partial x^2}[\rho S\omega^2\psi_T(x) - EI\frac{\partial^4\psi_T}{\partial x^4}(x)]dx(x)$$

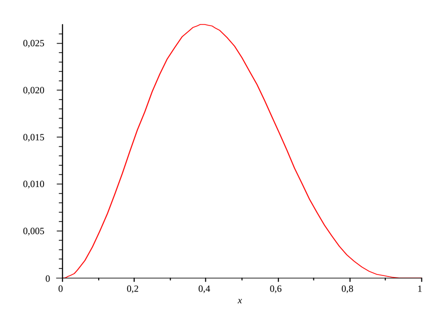
For practical reasons, two different patches, bonded to the opposite sides of the beam, can be used:

-a mass patch with a width:

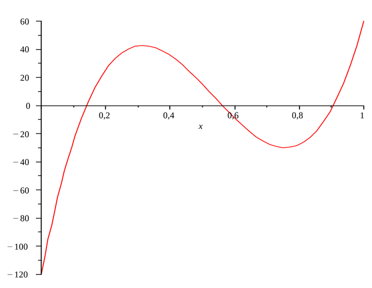
$$b_p(x) = \psi_T(x) = \frac{1}{2}x^2 - 5\frac{x^4}{a^2} + 10\frac{x^5}{a^3} - \frac{15}{2}\frac{x^6}{a^4} + 2\frac{x^7}{a^5}$$

-a stiffness patch with a width:

$$b_p(x) = \frac{\partial^4\psi_T}{\partial x^4}(x) = -\frac{120}{a^2} + 1200\frac{x}{a^3} - 2700\frac{x^2}{a^4} + 1680\frac{x^3}{a^5}$$



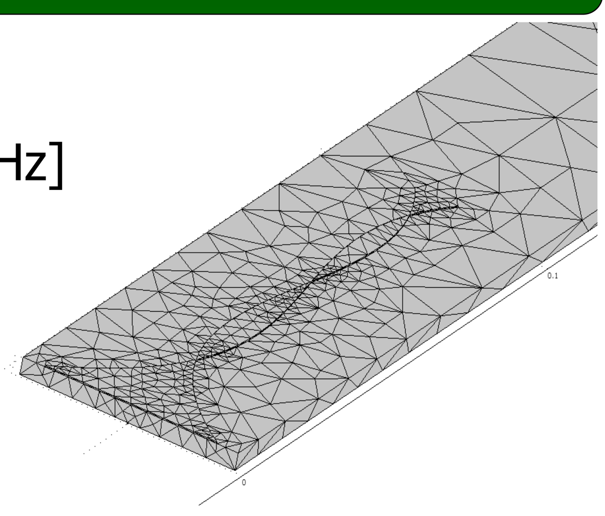
First shape

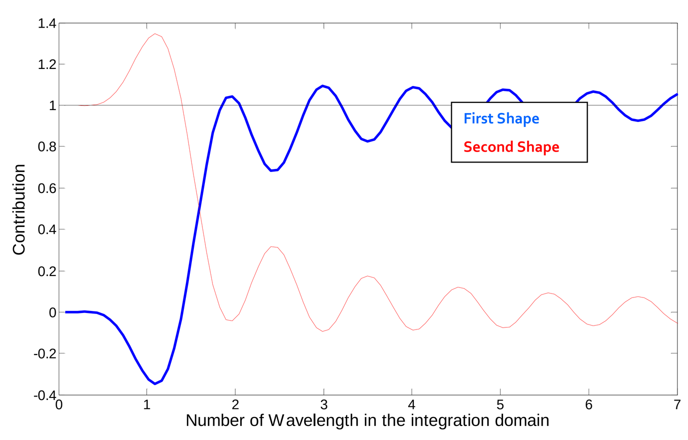


Second shape

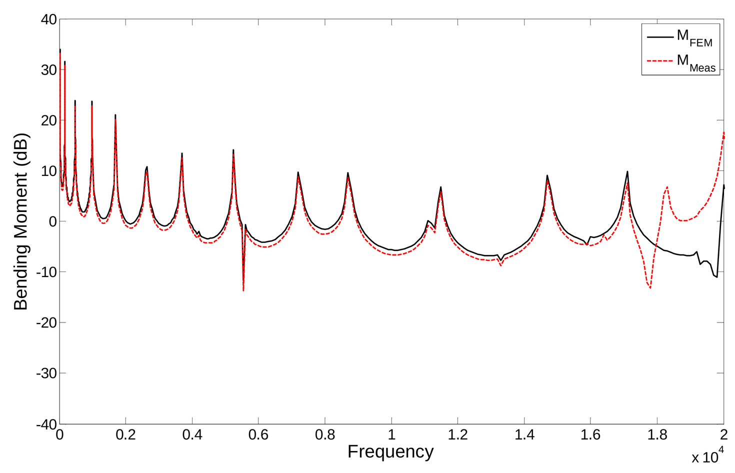
Numerical simulations

Use of a multi-physics FE software
In the frequency range of [1-25000Hz]



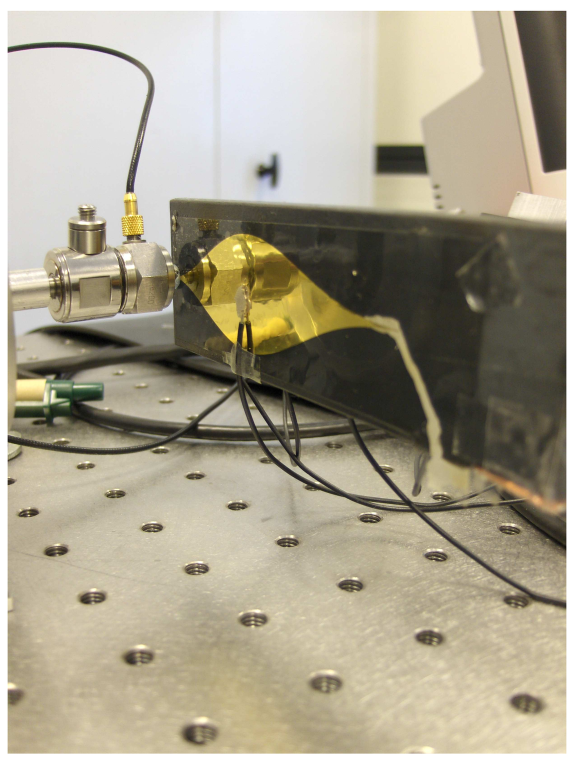


For small patches (in comparison with the natural wavelength), the stiffness patch delivers the entire useful signal.

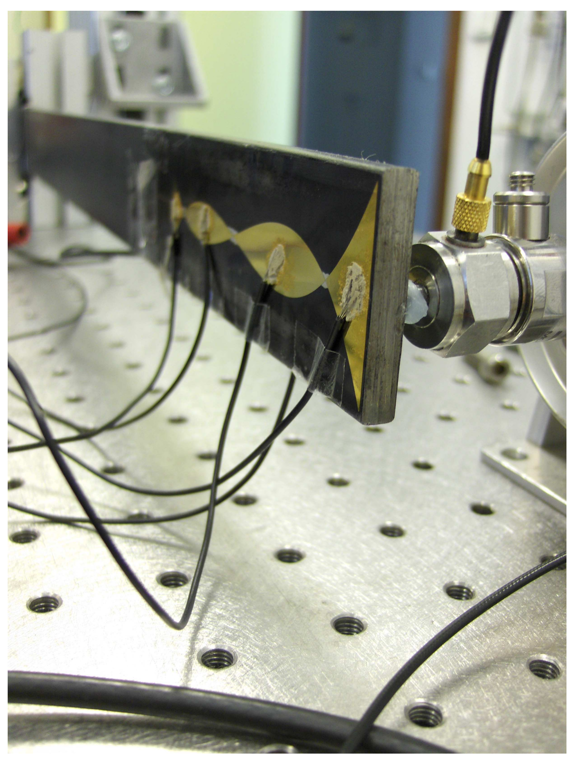


Identified forces are determined with a good accuracy, in a large frequency range [0-17000Hz].

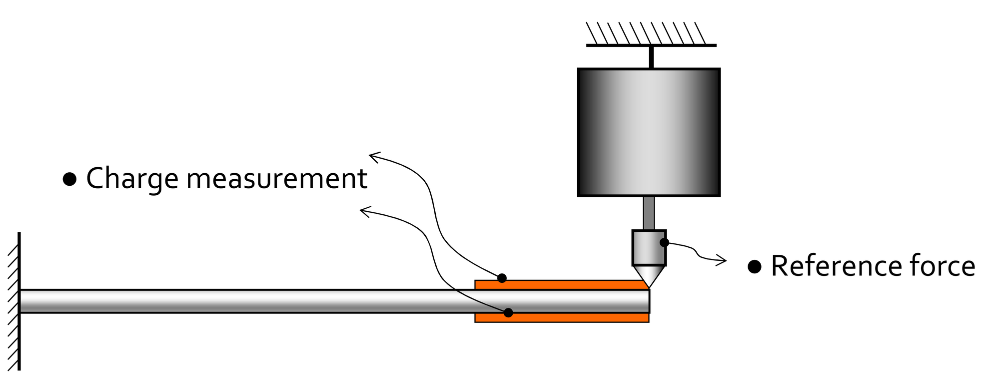
Experimentations



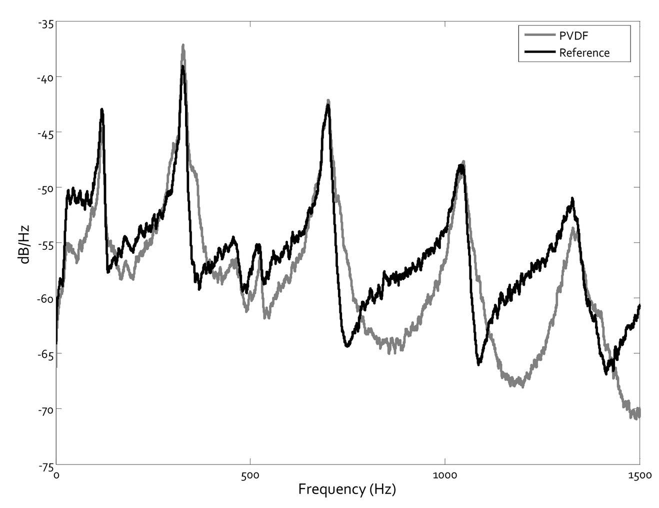
Mass Patch

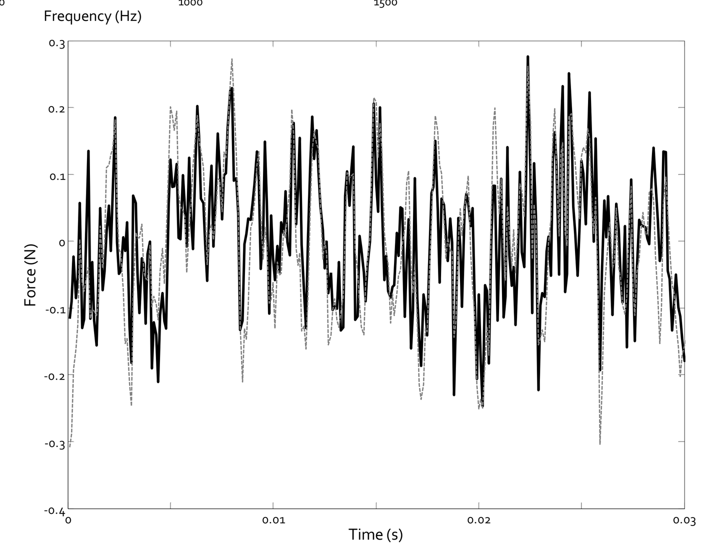


Stiffness Patch



At low frequencies, the frequency and time reconstructions give accurate results.





Conclusions

By comparison with other identification methods, this measurement technique presents a regularizing characteristic. Force and moment sensors are developed for boundary measurement in beam-like structures.

Distributed piezoelectric sensors for boundary force measurements in Euler–Bernoulli beams
Chesne S., Pezerat C. 2011 Smart Mater. Struct. 20
Design of PVDF sensors for shear force measurements in beams
Chesne S., Pezerat C. ISMA 2012 - Leuven, Belgique

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