Optimization Procedure for Identifying Constitutive Properties of High Speed Induction Motor

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Objectives

In order to predict the rotordynamics of a high speed induction motor in bending, an optimization procedure is proposed for identifying the constitutive properties especially those of the magnetic core. Modal parameters predicted by a finite element model based mainly on beam elements, and measured on an induction motor are included in modal error functions contained in a functional. The minimization of this functional by using the Levenberg-Marquardt algorithm permits extracting the constitutive properties **along** the magnetic core.

Squirrel Cage Rotor

- Assembly Structure:
  - ➔ Tie rods, Short-circuiting rods, Laminated stack.
- Prestressed Structure.
- Laminations without central hole.

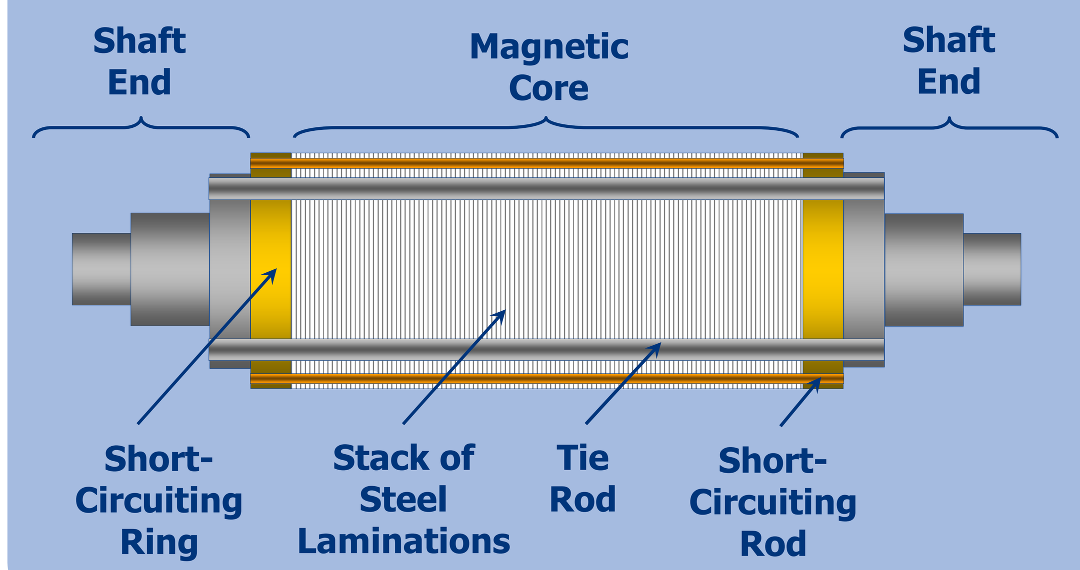

Figure 1. Diagram of a squirrel cage induction motor.

Kinetic Assumptions

Magnetic Core Modeling

- Short-circuiting rods are modeled as beams, of diameter D_{CC} (Fig. 3), whose neutral axes coincide with the neutral axis of the magnetic core.
- Tie rods can be seen as external tendons.
- Laminated stack is modeled as a distributed orthotropic material.

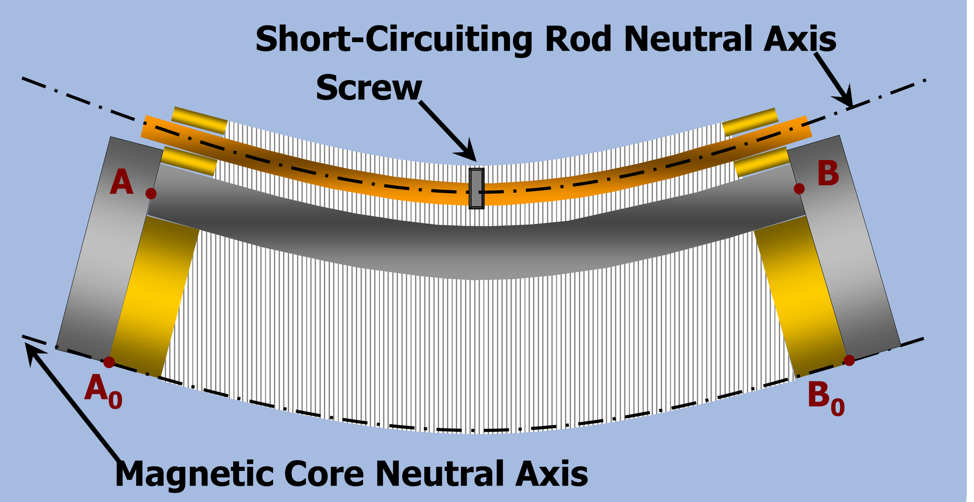

Figure 2. Short-circuiting rod in bending - Kinematic assumption.

Beam Finite Element Model

- Tie rods stiffness matrix at clamping nodes A_0 and B_0 (Fig. 2):

$$K_{TI}=\beta\cdot\frac{E_{TI}S_{TI}}{L_{TI}}\cdot e_{TI}^{2}\cdot\begin{pmatrix}0&0&0&0\\0&1&0&-1\\0&0&0&0\\0&-1&0&1\end{pmatrix}.$$

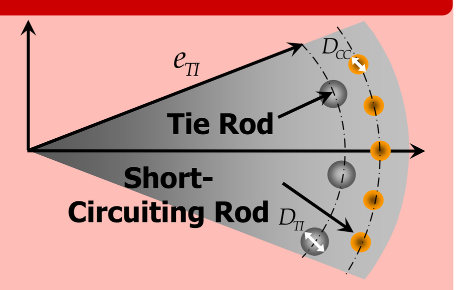

Figure 3. Cross section of the magnetic core.

Constitutive Property Distributions

- Distributions $x\in\Re^{n}, n=3h, h\in\mathbb{N}$, considered by splitting the stack into h sub-domains Ω_r such as $\left(x_p\right)_{p=3r-2,3r-1,3r}=\{E_r,G_r,\upsilon_r\}, r=1,\dots,h.$

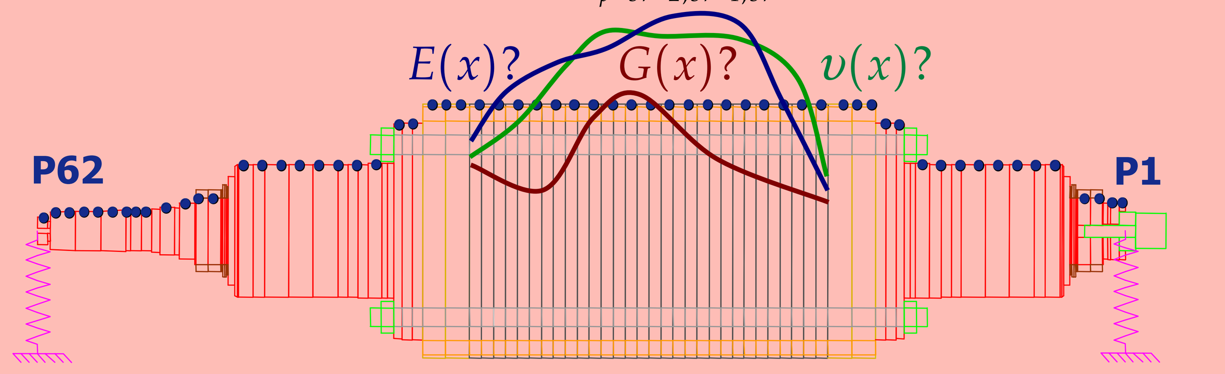

Figure 4. Finite element model. Blue points represent measurement points.

Modal Error Functions

- The difference between measured and predicted natural frequencies and mode shapes is quantified by four modal error functions, for each mode k, $k = 1 \dots m$:

$$F_k^{\omega}\left(x^i\right)=\frac{\tilde{\omega}_k^i}{\omega_k}-1,\quad F_k^{\varphi ED}\left(x^i\right)=\sum_{j=1,j\neq k}^{m}\left|MAC\left(\tilde{\varphi}_j^i,\varphi_k\right)-MAC\left(\varphi_j,\varphi_k\right)\right|,$$

$$F_k^{\varphi D}\left(x^i\right)=1-MAC\left(\tilde{\varphi}_k^i,\varphi_k\right),\quad F_k^{\varphi EC}\left(x^i\right)=\sum_{j=1}^{r}\left|\frac{\left(\tilde{\varphi}_k^i\right)_j}{\max_l\left(\left(\tilde{\varphi}_k^i\right)_l\right)}-\frac{\left(\varphi_k\right)_j}{\max_l\left(\left(\varphi_k\right)_l\right)}\right|.$$

Multi-Objective Function

$$F^{Tot}\left(x^i\right)=\left[\alpha^{\omega}F^{\omega},\alpha^{\varphi_D}F^{\varphi_D},\alpha^{\varphi_{HD}}F^{\varphi_{HD}},\alpha^{\varphi_{Ec}}F^{\varphi_{Ec}}\right]^{T},$$

$$\alpha=\frac{1}{\overline{F\left(x^0\right)}}.$$

➔ Weight coefficients based on the initial mean value, *i = 0*.

Levenberg-Marquardt Algorithm

- The difference is defined as a non-linear least squares functional, a global error norm:

$$f\left(x^i\right)=\frac{1}{2}\left\|F^{Tot}\left(x^i\right)\right\|^2=\frac{1}{2}\sum_{k=1}^{q}\left[F_k^{Tot}\left(x^i\right)\right]^2, q=4\times m.$$

- Optimization strategy consists in minimizing the difference between the measured and predicted modal data, at each iteration i:

$$\begin{cases}x^0,\lambda_0 \text{ given}\\ d_i=-\left(J\left(x^i\right)^T J\left(x^i\right)+\lambda_i I\right)^{-1}\cdot\nabla f\left(x^i\right), \text{ with } \nabla f\left(x^i\right)=J\left(x^i\right)^T F^{Tot}\left(x^i\right).\\ x^{i+1}=x^i+\rho_i d_i\end{cases}$$

where $J_{sp}=\frac{\partial F_s^{Tot}\left(x^i\right)}{\partial x_p}, p=1,\dots,n, s=1,\dots,q.$

Undamped Eigenvalue Problem

$$\begin{cases}\left(K-\lambda_k M\right)\tilde{\varphi}_k=0\\ \tilde{\varphi}_k^T M\tilde{\varphi}_k=1\end{cases}, \lambda_k=\tilde{\omega}_k^2, k=1,\dots,m,$$

where K and M are the stiffness and mass matrices respectively.

Analytical Eigen-Derivatives

$$\begin{cases}\frac{\partial\lambda_k}{\partial x_p}=\tilde{\varphi}_k^T\left(\frac{\partial K}{\partial x_p}-\lambda_k\frac{\partial M}{\partial x_p}\right)\tilde{\varphi}_k\\ \frac{\partial\tilde{\varphi}_k}{\partial x_p}=\sum_{j=1, j\neq k}^{m}c_j\tilde{\varphi}_j+c_k\tilde{\varphi}_k\equiv V_k+c_k\tilde{\varphi}_k\end{cases}.$$

➔ Nelson's Method

- Efficient Convergence.
- Less CPU-time Consuming.

The k^{th} eigenvector derivative requires only the k^{th} eigenvector.

Industrial Application

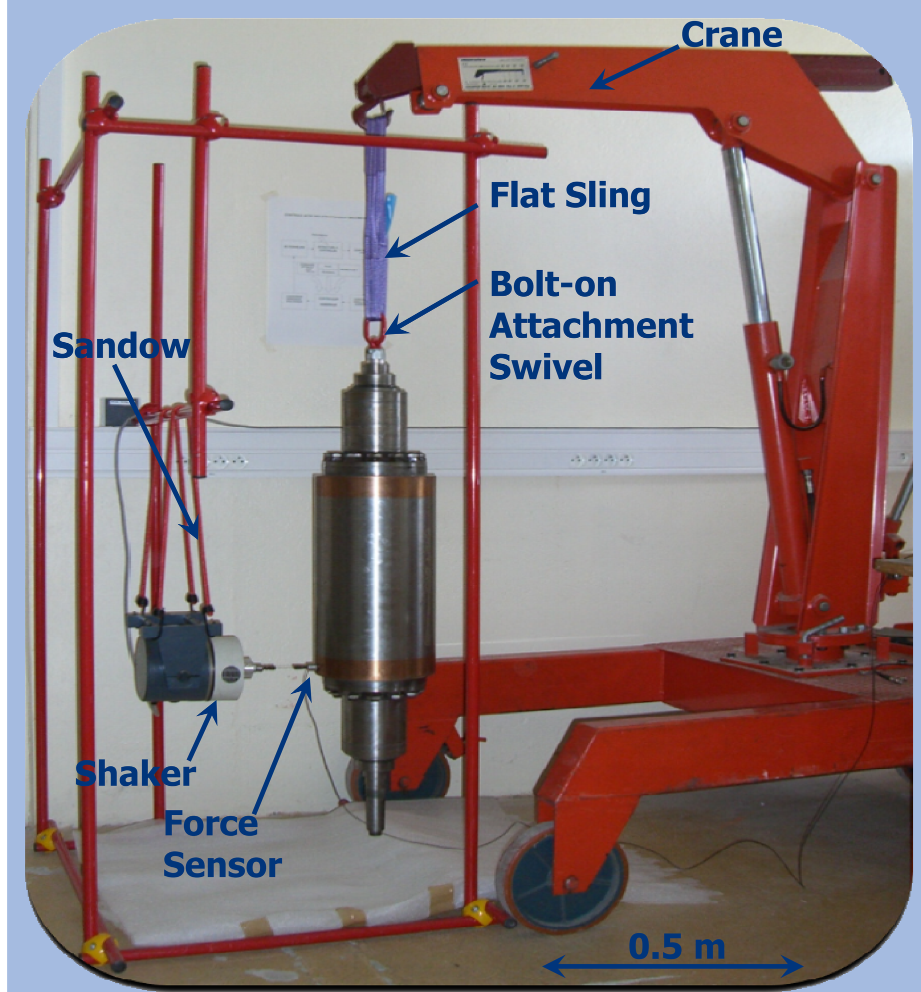

Figure 5. Experimental Setup.

Method Accuracy

➔Modal Testing $\{\varphi_k,\omega_k\}, k=1,\dots,4.$

- The global error norm decreases with the number of sub-domains.
- The global error norm obtained with $h = 7$ is height times lower (Fig. 6) than this one obtained with an isotropic assumption.
- The proposed method is more accurate than classical identification methods considering restrictive assumptions such as homogenous or isotropic material.

Table 1
First four bending measured and predicted natural frequencies obtained with 7 sub-domains.

	1st	2nd	3rd	4th
Measured (Hz)	856	1230	1713	2330
Predicted (Hz)	847	1263	1705	2315
Error (%)	1.05	2.68	0.47	0.64

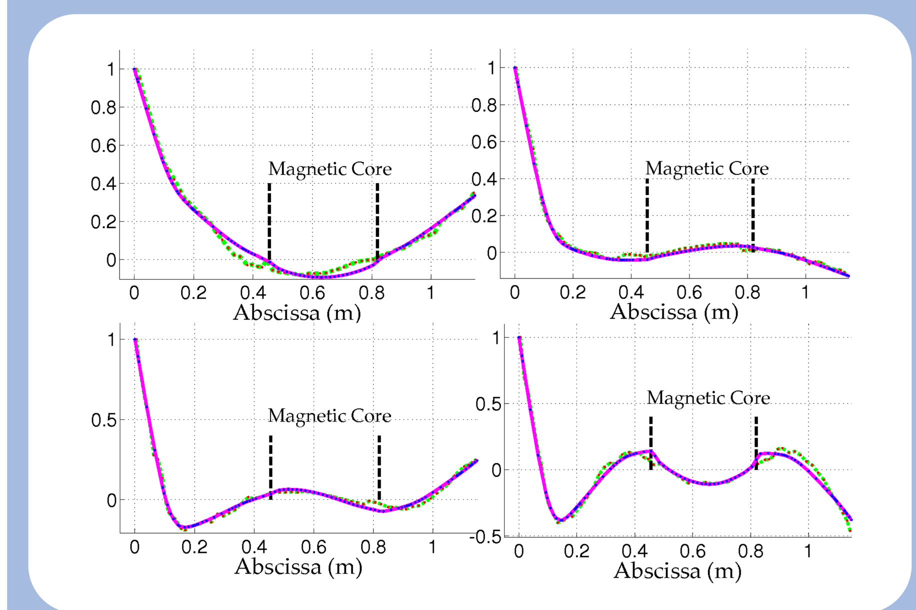

Figure 6. The dashed and solid line represent measured and predicted mode shapes respectively. Predicted mode shapes were obtained with 7 sub-domains.

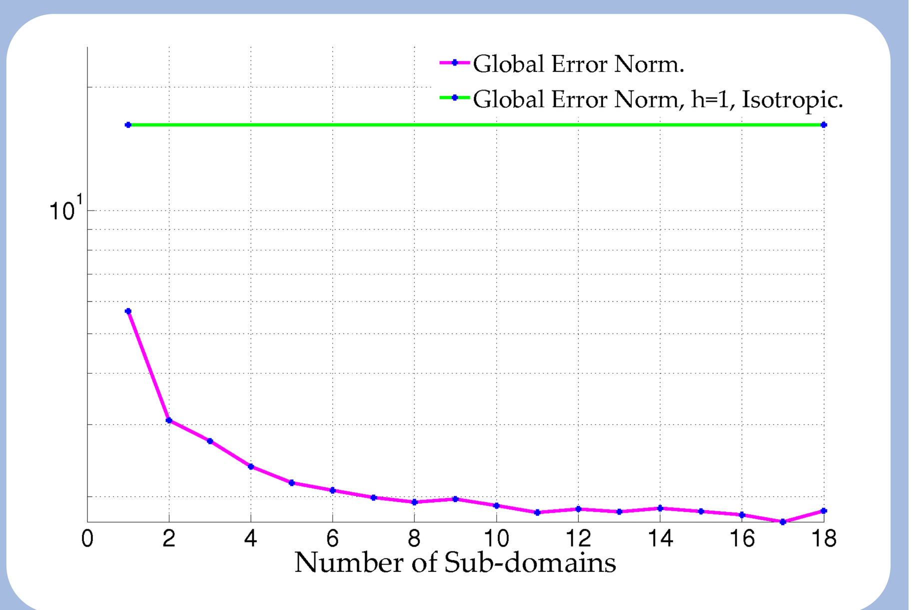

Figure 7. Evolution of the global error norm versus the number of sub-domains.

Sub-Domain Dependence

- Young's and Coulomb's modulus distributions tend to stabilize if $h > 7$ sub-domains.
- Distributions have constant values in the middle (55 % of the magnetic core length) of the magnetic core and decrease at its ends.

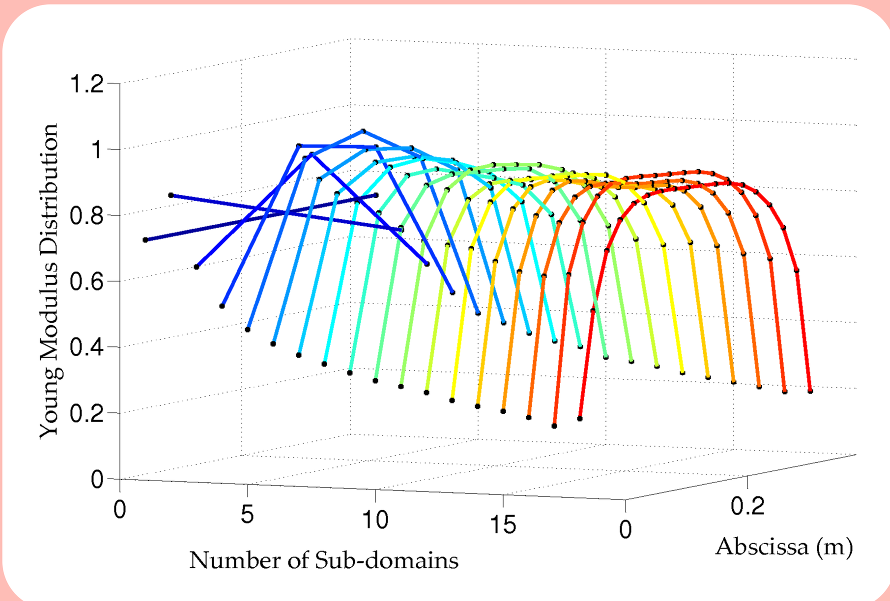

Figure 8. Normalized Young's modulus distributions.

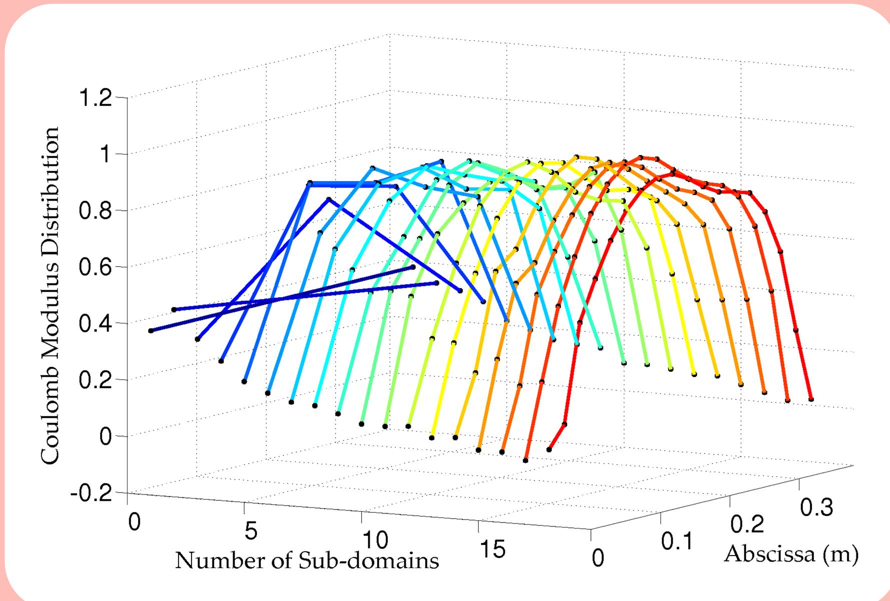

Figure 9. Normalized Coulomb's modulus distributions.

Conclusions

- Optimization procedure tested on an industrial rotor with complex design.
- Constitutive property distributions depend on the number of sub-domains used in the finite element (FE) modeling of the magnetic core.
- Distributions tend toward a particular shape versus the number of sub-domains: Constant values in the middle (55% of the magnetic core length) and Low values at its ends.
- The low level permits probably to consider the shear effect located close to the ends.
- This procedure is very useful for establishing an FE model based mainly on beam elements and therefore containing few degrees of freedom.

➔ Great advantage for predicting the rotordynamics, *i.e.* unbalance responses and transient responses.