

---

## The role of length scales in material failure

**Abstract:** Most materials exhibit markedly different behaviors at the micro-scale compared to the macro-scale, particularly when it comes to failure. At very small scales, solids tend to be significantly stronger than at the structural level. This difference is typically attributed to the presence of defects. While fracture mechanics provides an adequate framework for describing structural resistance in the presence of large defects, structures are rarely designed with intentional cracks. Most defects, whether introduced during manufacturing or as a result of wear over the material's lifetime, are relatively small and fall within a transitional scale. At this length scale, neither stress-based analysis nor traditional fracture mechanics are fully applicable.

The manuscript explores the critical role of length scales in material failure, offering new insights into the mechanical behavior of solids across different scales. The primary objective is to bridge the gap between theoretical fracture models and practical applications through the use of advanced computational techniques such as phase-field methods and atomic-scale simulations. By integrating multi-scale modeling approaches with experimental analysis, the study focuses on understanding fracture processes in brittle materials.

The manuscript presents several key findings that advance the understanding of fracture mechanics and material behavior across multiple scales. It establishes correlations between fracture toughness and other material properties, such as tensile strength in various fracture modes. It also reveals that material length scales differ from the size of instabilities observed in complex geometries. Furthermore, the study shows that the limiting velocities of cracks in dynamic fractures are linked to the average stiffness around the crack tip, where the finite-sized, regularized stress zone helps align simulations with experimental observations. The research demonstrates how different length scales in atomic structures manifest in elasticity, plasticity, and fracture, providing new insights into material behavior. Additionally, the study introduces failure criteria inspired by atomic-scale processes in silicate glasses, which are crucial for accurately predicting both their fracture and ductile responses. The critical role of diffuse damage in differentiating between free surface energy and fracture surface energy is also emphasized. Finally, the study shows that periodic beam lattices possess a unique fracture toughness determined by their elementary structure and tensile strength, while establishing that Cosserat theory is both necessary and sufficient for optimizing the mechanical performance of these structures.

The implications of these findings are profound, offering new pathways for optimizing fracture-resistant materials. The research suggests that local toughness can be homogenized. By applying advanced optimization techniques, the global resistance of materials can be significantly enhanced, paving the way for the development of customized, fracture-resistant mechanical metamaterials tailored to specific applications.

**Keywords:** Regularization length, Toughness, Phase-field fracture, Atomic scale simulations, Silicate glass, Beam lattice

---