A hybrid analytical / extended finite element method for direct evaluation of stress intensity factors

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Introduction

- Stress intensity factors $K_I$ and $K_II$ are key parameters for fracture mechanics
- Need for a robust evaluation
- No post-processing, direct evaluation
- Mesh independence
- Optimal convergence
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Crack displacement fields

Let us consider a homogeneous body with isotropic elastic behavior, and a 2D setting, the displacement field $u$ is conventionally represented by its complex writing, $u = u_r + iu_\theta$. It was expanded by Williams [4] for a straight crack as a double series

$$u(r, \theta) = \sum_{i=1,II} \sum_n c_i^I(r, \theta)$$

with

$$\phi_i^I(r, \theta) = r^{n/2} \left( \kappa e^{(4n-1)\theta/2} + \frac{n}{2} - (1-i) e^{-n\theta/2} \right)$$

$\phi_i^II(r, \theta) = r^{n/2} \left( \kappa e^{(4n-1)\theta/2} + \frac{n}{2} e^{(4n-1)\theta/2} - (1-i) e^{-n\theta/2} \right)$

where $\kappa$ is Kolossov’s constant, namely, $\kappa = (3-\nu)/(1+\nu)$ for plane stress or $\kappa = (3-4\nu)$ for plane strain conditions, $\nu$ being Poisson’s ratio.

- $n = 0$: translation
- $n = 2$: rotation and T-stress
- $n = 1$: usual asymptotic fields
- $n \geq 3$: sub-singular fields

Results

- Displacement field in m
- $K_{IC} = f(\epsilon_{tens}) \sigma_{th} \sqrt{a} = 2.98 \text{ MPa}\sqrt{m}$
- Region of analytical model $\Omega_{an}$
- Region of fracture elements $\Omega_{fract}$
- Coupling region $\Omega_{coup}$
- Number of terms in the analytical model $n_{max}$

Conclusions and perspectives

- Hybrid analytical / extended finite element method
- Accuracy and robustness wrt. geometrical parameters
- Quasi-optimal convergence of SIFs
- Crack propagation
- Analytical solutions for cohesive cracks
- Digital image correlation

Bibliography


