Advanced Chebyshev expansion for identification on continuous structures

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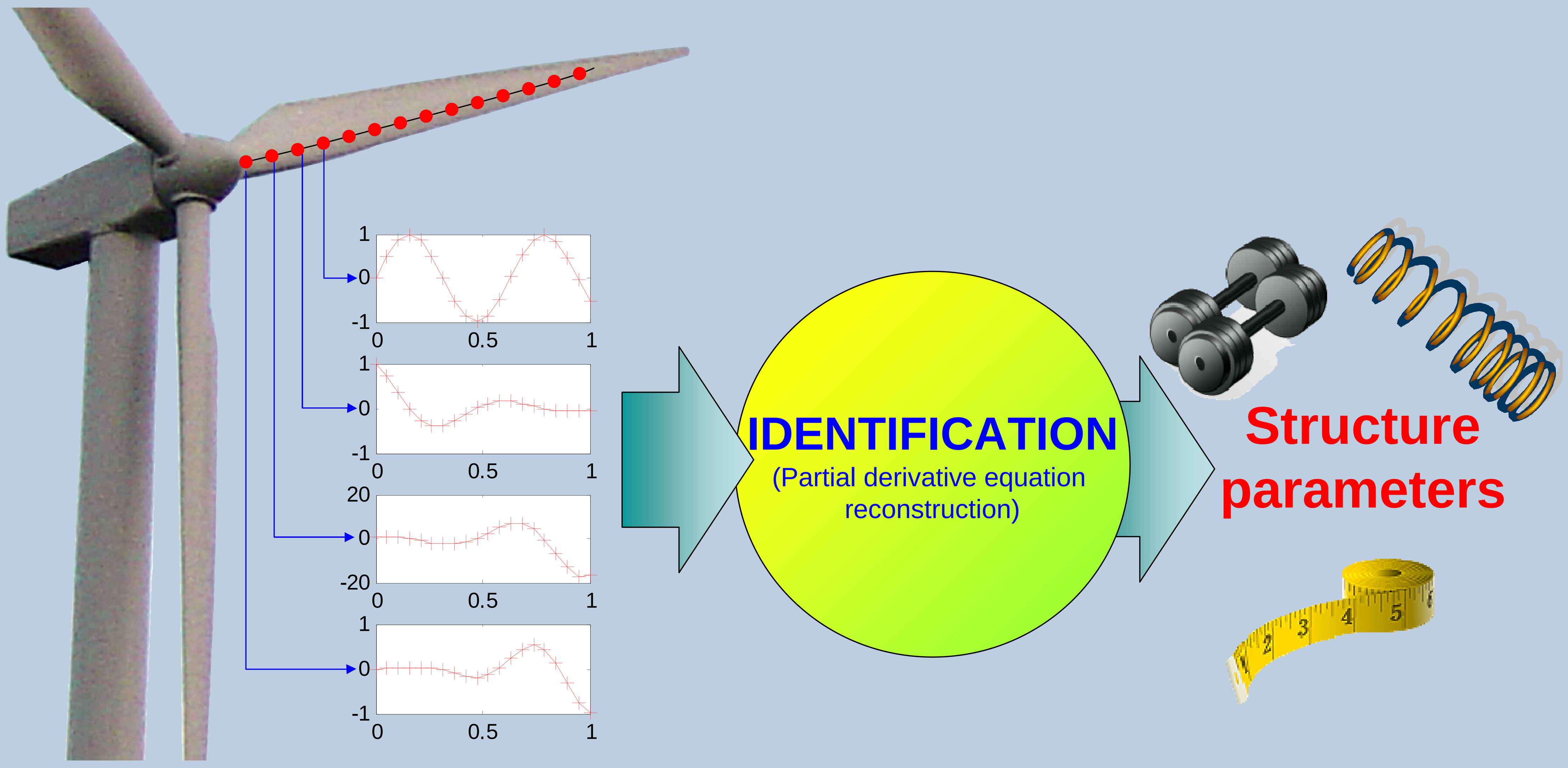
Introduction

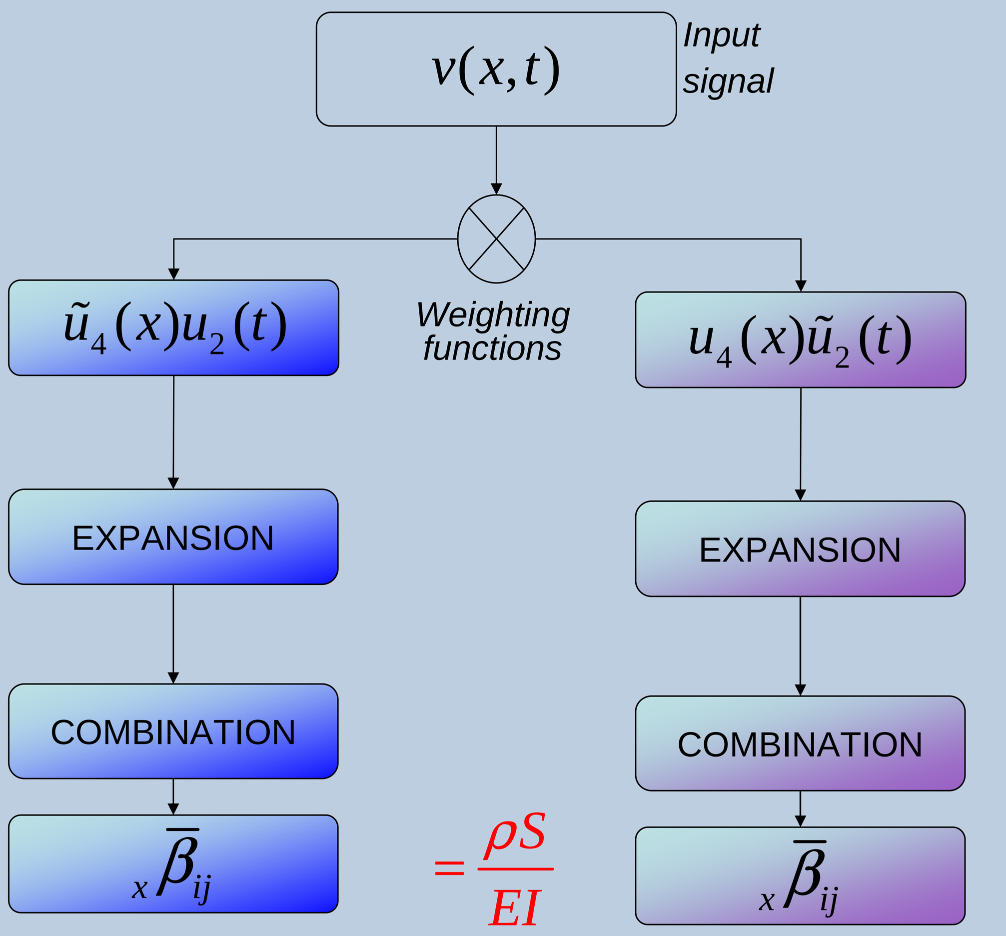


There are strong needs for health-monitoring of continuous, multi-dimensional structures as bridges,aircraft wings or wind turbine.
The present contribution is dedicated to the use of Chebyshev orthogonal functions to expand the response of a system.
Based on this expansion, a novel computation of the partial derivatives is presented.
As explained in [1, 2], the purpose of this work is to estimate a structure parameter, which does not depend on the structure environment. Therefore, the partial differential equation of structure motion is reconstructed numerically.

Problem formulation

The identification process is splited in 4 steps :

- the recorded signal (displacement) $v(x,t)$ is multiplied by weightning functions : $\tilde{u}_4(x)u_2(t)$ or $u_4(x)\tilde{u}_2(t)$
- the obtained data are expanded on the Chebyshev orthogonal basis,
  writing $f(x,t) \approx \sum_{i=0}^{N}\sum_{j=0}^{M} \lambda_{i,j} P_i(x)P_j(t)$
- the $\lambda_{i,j}$ are combined (refer to *Differentiation method*) in order to obtain the expansion of the partial derivatives, e.g.
  $$u_4(x)u_2(t)\frac{\partial^4 v}{\partial x^4}(x,t) \approx \sum_{i=0}^{N}\sum_{j=0}^{M} {}_x\bar{\beta}_{i,j} P_i(x)P_j(t)$$
- the partial differential equation is reconstructed in order to obtain $\rho S/EI$



$$u_4(x)u_2(t)\frac{\partial^4 v}{\partial x^4}(x,t) = \frac{\rho S}{EI} u_4(x)u_2(t)\frac{\partial^2 v}{\partial t^2}(x,t)$$

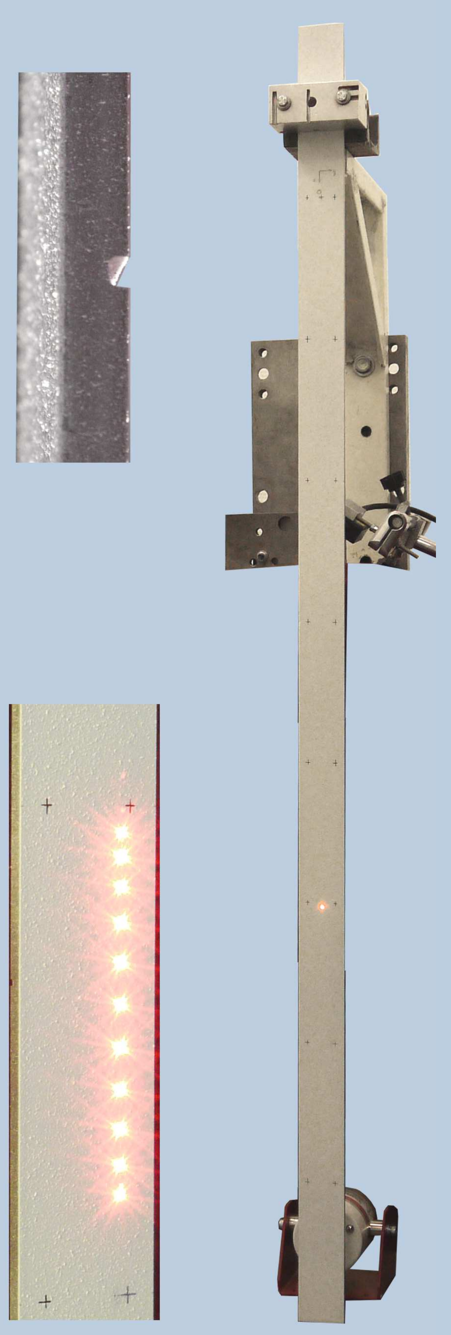
(ρ : density, E : Young modulus, S : cross-section area and I : flexural inertia)

Differentiation method

- Chebysev scalar product :
  $$ {}_x\bar{\beta}_i = < u \cdot f', T_i > = \int_{-1}^{1} f'(x)u(x)T_i(x)w(x)dx$$
- Integrating by part:
  $$ {}_x\bar{\beta}_i = -\int_{-1}^{1} f(x)(u(x)T_i(x)w(x))'dx$$
- We can find:
  $$(u(x)T_i(x)w(x))' = w(x)\sum_{j=i-1}^{j=i+1} \alpha_j \tilde{u}(x)T_j(x)$$
- Then
  $$ {}_x\bar{\beta}_i = \sum_{j=i-1}^{j=i+1} \alpha_j < \tilde{u}\cdot f, T_j(x) >$$

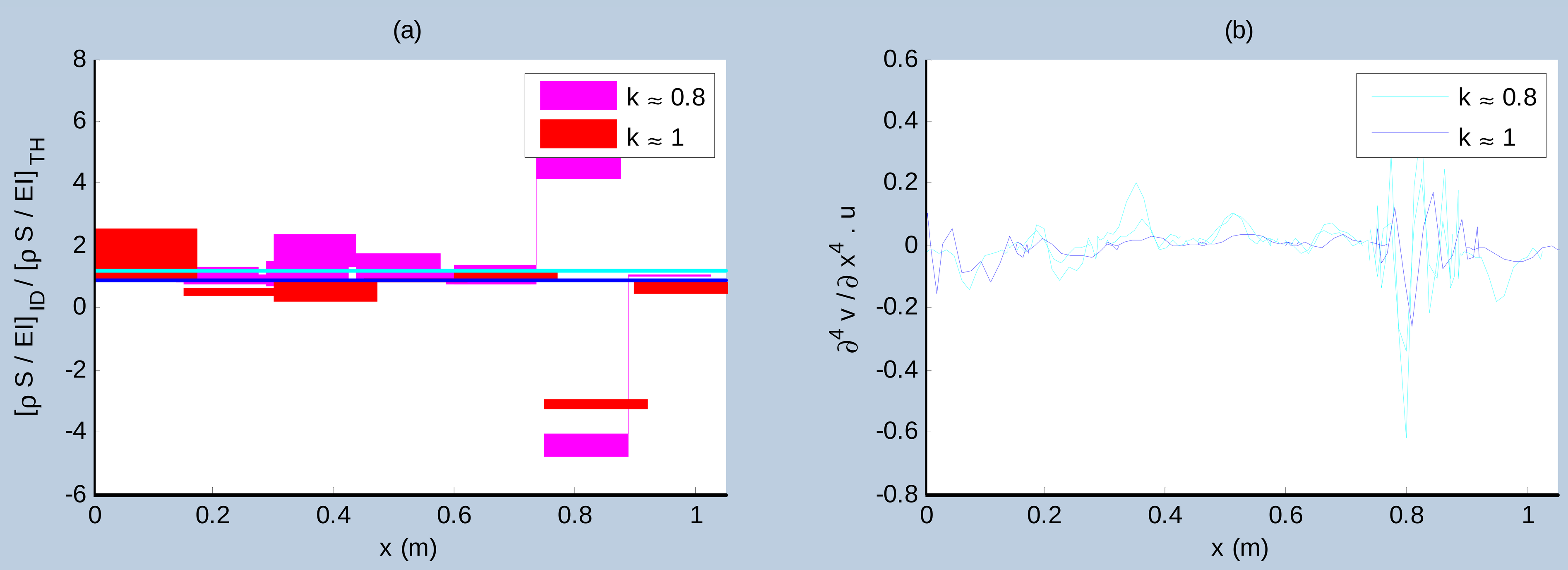
Experimentation

The displacement of this beam is reconstructed with selected time samples and space positions using a laser vibrometer (PSV400). Here, we study the forced response of this cantilever beam. 1.1m long in which a crack is imposed (a notch 3mm in width and 5mm in depth). This figure shows the beam, the imposed crack and laser measurements at different positions. A sensor array provides 16 measurements at 16 different sample positions.



Results

In this figure, (a) is computed $\rho S/EI_{ID}/\rho S/EI_{TH}$ and (b) is computed $(\partial^4 v/\partial x^4 \cdot u)$ for the cantilever beam. Results are based on a experimental data (excitation frequency equal to 1471Hz). On the left, the light and dark grey dashed lines correspond to the mean values (for $k \approx 0.8$ and $k \approx 1$ respectively).



These results demonstrate an alternative method for damage location. Indeed, with a restricted number of samples, it was shown that $\rho S/EI$ and the fourth derivative $\partial^4 v/\partial x^4$ can be computed. The parameters studied are very sensitive to damage (a discontinuity). $\rho S/EI$ becomes negative when the continuity assumption is no longer valid. The damage can also drastically increase the $\rho S/EI$ value.

References

[1] C Chochol, S Chesne, and D Remond. Identification à temps continu sur structure continue. In *CFM conference*, 2011.

[2] C Chochol, S Chesne, and D Remond. Advanced chebyshev expansion for identification of smart structures. In *SYSID conference*, 2012.