Timestepping schemes based on discontinuous Galerkin methods

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In nonsmooth dynamical systems, state trajectories may be classified in impulsive and non-impulsive periods. There are two types of integration schemes which can handle both types of motion. On the one hand, event-driven methods are highly recommendable in non-impulsive periods e.g. because of their high integration order. However, they cannot represent an infinite accumulation of impacts. On the other hand, classic timestepping schemes are applicable in impulsive periods but they are of integration order one in both impulsive and non-impulsive periods.

An enhancement of classic timestepping schemes based on discontinuous Galerkin methods and splitting of non-impulsive and impulsive force propagation has been presented recently [1]. It benefits from the advantages of event-driven and classic timestepping methods and avoids their disadvantages. A typical representative is the explicit trapezoidal-rule type timestepping method, called $D-$ linear timestepping method.

In this talk, we first revisit the basic idea of the $D-$ linear timestepping method in particular. Thereby we assume that ansatz and test functions for the velocity might have jumps across discretization intervals, and that they are of higher integration order inside discretization intervals. Further by splitting contact and impact reactions, contact forces benefit from higher order ansatz functions and impact reactions yield local integration order one. The order of convergence of position, velocity and impulse trajectories is discussed in comparison to the classic Moreau-Jean timestepping method concerning free flight and accumulation of impacts in the bouncing ball example.

Second, we apply the $D-$ linear timestepping method to a multi-dimensional example, i.e. the impacting slider-crank mechanism without friction [3]. Thereby, we evaluate contact forces as in the original paper [1] on acceleration/force level and impact reactions on velocity/impulse level using the prox-formulation [2].

In a third step, we propose also a velocity/force level evaluation of contact forces. This description is compared to the acceleration/force level description using the impacting slider-crank mechanism. In addition, Coulomb friction is added to the discretization scheme with contact descriptions on velocity/force level.

At the end of the talk, a conclusion and an outlook is given.

References

[1] Thorsten Schindler and Vincent Acary. Timestepping schemes for nonsmooth dynamics based on discontinuous Galerkin methods: definition and outlook. *Math Comput Simulat*, to appear 2013.

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[3] Svenja Schoeder, Heinz Ulbrich, and Thorsten Schindler. Discussion of the Gear-Gupta-Leimkuhler method for impacting mechanical systems. *Multibody System Dynamics*, to appear 2013.