Proposal for PhD thesis: LABEX CELYA

Nonlinear interactions of mass-in-mass cells in a lattice of acoustic metamaterial

1 Preliminary discussions

Some researches have been carried out on which consider the dynamics of mass-in-mass cells for creation of suppression band of systems. Considering a single linear unit of such systems can lead to the negative effective mass depending on the frequency of the deriving excitation [1]. In this case and according to the Newton's law, the equivalent response force will be in opposite direction of the deriving excitation and so the response of the systems will decrease. This effect is more pronounced when the deriving frequency approaches the resonance frequency of the inner mass [2]. These studies have been extended to consideration of a lattice [3] showing that including nonlineaities the band gap of suppression would increase [2]. Lazarov and Jensen [4] considered an infinite chain of globally linearly coupled particles where each global particle is attached to another local mass in nonlinear (or linear) manner. This system can be represented as mass-in-mass cells. They showed that it is possible to create some band gaps in such systems. For the case of local linear attachments, the gap is located around the frequency of local particles while the position of the band gap can be shifted in the nonlinear case depending on the type of the nonlinearity. Later on, Lazarov and Thomsen [5] studied a system which can be related to a one dimensional model of materials with included nonlinearities. In fact the system was a mono-dimensional lattice with periodically coupled linear and then nonlinear particles. They have seen that it is possible to alter low frequency properties of such system using high frequency excitations. In the following, we will discuss about our proposal and its different steps via connecting to the idea of mass-in-mass cells.

2 The proposal

Our considered systems is composed of a nonlinear metamaterial lattice which includes coupled mass-in-mass cells. The global inter cell reactions and local coupling terms are nonlinear in general. We also intend to consider the nonlinear springs for the inner mass as well. The proposed PhD subject aims at preparation of design tools for spacial acoustics metamaterials. The proposal contains following steps:

Analytical step Fast and slow dynamics [6] of the lattice should be traced; in detail: the slow invariant manifold (SIM) of the system should be detected at fast time scale while all characteristic points around the SIM should be clarified at the slow time scale. These points read as system equilibria and singularities standing for periodic (stable or unstable) and strongly modulated responses [7,8]. The latter correspond to strong interactions, bifurcations and energy exchanges between different modes of the lattice [9]. The detection of characteristic points could demand exploiting algebraic methods for tracing roots of polynomials (eg. see [10]). The threshold of the filtering pass will be adjusted and analytical design tools for tuning nonlinear parameters of the lattice for this aim will be prepared. All of above mentioned developments will be carried out for a lattice which each cell of it contains a single or several inner nonlinear mass(es). For the case of cells with several inner masses, we will consider a general case where each inner particles possesses different properties such as nonlinear rigidities. This will permit to study the possibility of broadening the filtering pass by using several inner masses with different properties.

Numerical step All analytical developments in previous steps should be verified by numerical tools: Developed [11, 12] or adopted tools [13] for instance continuation or shooting techniques will be endowed for tracing all equilibrium points. Stabilities of such points and detection of Neimark-Sacker bifurcations [14] should be pinpointed as well.

Experimental step A lattice of mass-in-mass cells (with single and/or at least two inner masses) will be designed and fabricated by the 3D-printing facilities of the LTDS team in ENTPE (ProJet 3510SD of 3D Systems Europe Ltd.). The system will be tested on the 6 axes vibration test system of Equipex PHARE located at LaMCoS or on the shake table of the ENTPE-LTDS with different and varying deriving excitations. Analytical, numerical and experimental results will be compared in order to validate the established models.

3 Directors of the thesis

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